Statistics 104

Midterm Examination 2 Solutions

1. (10 points) Indicate which of the following statements are true and briefly, for each of the others, show why they are false. You may simply correct the given statement as a way of showing why.

   a) (2 points) Two cities are considering putting initiatives on the ballots in the next election to raise taxes for the purpose of increasing the size of their police forces to combat crime. Each city would like to get an idea as to the level of support for the tax increases in each of their communities. Each city plans to call 500 randomly selected voters and ask them whether they would support the initiative or not. One city has 100,000 residents and the other has 400,000. The standard error for the proportion of voters in favour of raising taxes is approximately the same for both cities.

      True. Since the sample size is a small fraction of the population sizes, the finite population correction will make little difference.

   b) (2 points) Blocking in designed experiments is mainly a way of reducing the bias in the comparisons of the treatments.

      False. Blocking is a way of reducing the variation in the comparison of treatments.

   c) (2 points) A university has 2000 male and 500 female faculty members. The affirmative action committee want to poll the opinions of a random sample of faculty members. The committee randomly selects 50 individuals from an alphabetized list of female faculty members and 200 individuals from a similar list of male faculty members, using methods discussed in class. This is an example of a simple random sample.

      False. Stratified random sample

   d) (2 points) A good statistic to use to estimate a parameter is one with high bias and high variability.

      False. Small bias and small variability

   e) (2 points) As the sample size increases, the sample variance s^2 should get closer to zero.

      False. s^2 should get closer to σ^2, the population variance.

2. (11 points) You have been asked to design a study examining student opinions about whether Harvard should switch to a quarter system. For simplicity, a simple random sample of students is to taken.

   a) (4 points) How big a sample would be needed if the standard error for the sample proportion of students in favour of the change is to be no more than 0.025?

   Want $\sqrt{\frac{p(1-p)}{n}} \leq 0.025$ for any choice of p. Since the standard deviation of $\hat{p}$ is maximized by $p = 0.5$, this gives

$$\sqrt{\frac{0.5 \times 0.5}{n}} \leq 0.025 \Rightarrow n \geq \frac{0.25}{0.025^2} = 400$$

   b) (4 points) Suppose that if the sample proportion $\hat{p}$ is greater than 0.6, the students will be classified as being in strongly in favour of the proposal. What is the probability of this happening if the true proportion of students in favour of the proposal is 0.55 and a sample of 625 students was taken.

$$\sigma_{\hat{p}} = \sqrt{\frac{0.55 \times 0.45}{625}} = 0.0199$$

$$\begin{aligned} P\left[\hat{p} \geq 0.6\right] &= P\left[\frac{\hat{p} - 0.55}{0.0199} \geq \frac{0.6 - 0.55}{0.0199}\right] \\ &\approx P\left[Z \geq 2.51\right] = 1 - 0.994 = 0.006 \end{aligned}$$

   c) (3 points) Suppose that if the study is performed with the sample size you calculated in part a), one half of the sample is comprised of undergraduate students and the other half is graduate students. What would be the standard error for the sample proportion of undergraduate in favour of the proposal?

   With a sample of half the size the standard error will increase by a factor of $\sqrt{2}$ as

$$\sqrt{\frac{p(1-p)}{n/2}} = \sqrt{2}\sqrt{\frac{p(1-p)}{n}}$$

   The maximum this can be is $0.025 \times \sqrt{2} = 0.0354$

3. (17 points) As part of a study to compare the physical education programs at two Canadian schools, running times (in seconds) over a set distance were recorded for random samples of grade 6 students. Summary statistics are given below.

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Descriptive Statistics

Variable      School    N     Mean    Median   StDev  SEMean
Time        Glooscap   12   13.312    13.080   0.991   0.286
           Coldbrook   13   12.228    12.010   0.990   0.275
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a) (4 points) Give a 95% confidence interval for the mean running time at Glooscap Elementary, assuming the observations are approximately normally distributed.

$n = 12 \Rightarrow \text{df} = 11 \Rightarrow t^* = 2.201$

CI = $13.312 \pm 2.201 \times 0.286 = 13.312 \pm 0.629 = (12.683, 13.941)$

b) (4 points) Is there any real difference in running times, on average, between the two schools? State appropriate null and alternative hypotheses and test at the $\alpha = 0.05$ level, assuming the observations are approximately normally distributed.

$H_0 : \mu_G = \mu_C$ versus $H_0 : \mu_G \neq \mu_C$

If you assume not equal variances:

$$t = \frac{13.312 - 12.228}{\sqrt{\frac{0.991^2}{12} + \frac{0.990^2}{13}}} = 2.73, \qquad \text{df} = \min(11,12) = 11 \Rightarrow t^* = 2.201$$

If you assume equal variances

$$s_p^2 = \frac{11 \times 0.991^2 + 12 \times 0.990^2}{11 + 12} = 0.9815$$

$$s_p = \sqrt{0.9815} = 0.991$$

$$t = \frac{13.312 - 12.228}{0.991\sqrt{\frac{1}{12} + \frac{1}{13}}} = 2.73, \quad \text{df} = 23 \Rightarrow t^* = 2.069$$

Since $t_{\text{obs}} > t^*$, reject H_0 and conclude that the average running times are different.

c) (2 points) Calculate or give bounds for the *p*-value for the test you calculated in part b).

If you assumed unequal variances, $2.718 \leq t_{obs} \leq 3.106$, which implies $0.01 \leq p\text{-value} \leq 0.02$.

If you assumed equal variances, $2.500 \leq t_{obs} \leq 2.807$, which implies $0.01 \leq p\text{-value} \leq 0.02$.

d) (2 points) State in terms that a general person would understand (e.g. somebody who hasn't taken a statistics course) what conclusions you can make about the differences in the mean running times for the two schools.

The data suggests that the average running time for children at Coldbrook is lower than that for children at Gooscap. The best guess for the difference in average running time is 1.084 seconds.

d) (2 points) In the test statistic you calculated in part b), what assumption did you make about the variances of the running times for the two schools?

Has to match with your answer to a)

e) (3 points) Gooscap Elementary children were taught by a specialist sprint coach. Do we have evidence that the running times of children are faster when coached by a specialist running coach than when coached by a generalist physical education teacher? To what extent does a study like this answer such a question?

There is really no evidence to support this. First the running times were worse at Gooscap. However this study really doesn't answer the question. First we don't know the effect of the coaching. It may have been that the Gooscap children showed greater improvement, even though they are still slower on average. Also we only have one observation on each treatment. A more extensive study would need to be done, involving multiple schools, specialist coaches and regular physical education teachers.

4. (12 points) Suppose that a faculty committee of three people is to be chosen at random from the following list of ten people

Blanchet (0)	Irwin (1)	Izem (2)	Kou (3)		
Chernoff (4)	Dempster (5)	Liu (6)	Meng (7)	Morris (8)	Rubin (9)

Based on the following sequence of random digits

44592 07511 88915 41267 16853 84569 79367 32337 03316

a) (4 points) Give a random committee assignment. Describe how you used the random digits to pick the three people

Assign 0 to Blanchet, 1 to Irwin, …, and 9 to Rubin as above. This gives a committee of

Chernoff (4), Dempster (5), Rubin (9). The second 4 in the list is skipped

b) (4 points) Suppose that it was found out later, that the committee should contain one person from the first row (junior faculty) and two people from the second row (senior faculty). Give a random committee assignment under this constraint. Again describe how you used the random digits to pick the three people.

One approach would keep the same number assignment as above. Start with selecting the person from the first row, which would be Izem (2), if you started at the beginning of the random digits. Then pick the two people from the second row, starting with the digit after the first 2. That leads to picking Meng (7) and Dempster (5).

Note that there are many different schemes that would lead to an equivalent stratified sampling scheme. For example the faculty in the first row could be coded

Blanchet (0,4) Irwin (1,5) Izem (2,6) Kou (3,8)

c) (4 points) What is the probability that the drawn committee (as selected by the mechanism used in part a)) will contain nobody from the first row?

$$\begin{aligned} P\left[\text{all from } 2^{\text{nd}} \text{ row}\right] = P&\left[\text{first person from } 2^{\text{nd}} \text{ row}\right] \\ &\times P\left[\text{second person from } 2^{\text{nd}} \text{ row} \middle| \text{first person from } 2^{\text{nd}} \text{ row}\right] \\ &\times P\left[\text{third person from } 2^{\text{nd}} \text{ row} \middle| \text{first two from } 2^{\text{nd}} \text{ row}\right] \\ = \frac{6}{10}\times\frac{5}{9}\times\frac{4}{8} = \frac{1}{6}& \end{aligned}$$