

## Statistics 104 - FALL, 2004 — Assignment 4

Due Monday, November 1, 2004.

**Reminder:** The first midterm exam is on Wednesday, October 27th. **The exam will take place in Lowell Lecture Hall during the normal class time.**

### Readings (Moore and McCabe)

- Sections 4.4, 4.5, Chapter 3.

### Written Assignment (Moore and McCabe)

- **MM:** 4.66, 4.79, 4.80, 4.81, 4.82, 4.83, 4.84, 4.96, 4.102, 4.110. **For the odd numbered problems, you must show your work to get any credit. Of course, showing you work for the even numbered problems is a good idea as well.**

### Additional written problem

The following table gives the proportions of people of each religion and occupational group in a fictitious Southeast Asian country.

	Farmer	Merchant	Civil servant	Laborer
Moslem	.21	.06	.06	.09
Buddhist	.42	.08	.03	.05

1. Copy the table and put on the marginal probabilities for each religion and occupational group.
2. Calculate the conditional probability that a person is Buddhist given that he is a farmer,  $P(\text{Buddhist} \mid \text{farmer})$ , and the conditional probability that a person is Moslem given that he is a farmer,  $P(\text{Moslem} \mid \text{farmer})$ . Do they add up to 1? Should they?
3. Calculate  $P(\text{farmer} \mid \text{Buddhist})$  and  $P(\text{farmer} \mid \text{Moslem})$ . Do these add up to 1? Should they?
4. In this country, is religion independent of occupation? Explain why or why not, referring to a (simple) calculation from the table.

### Challenge Problem: Markov Chains (from last week)

*As usual this is completely optional, with modest extra credit attached.*

Why did the chicken cross the road?  
To get to the other side.

A chicken starts out on the north side of a road. Every minute, it either crosses to the other side or stays where it is. If it is on the south side of the road, it has probability 0.30 of crossing to the north side, and otherwise it stays on the south side. If it is on the north side, dazzled by the southern sun, it has probability 0.50 of crossing to the south side. Being a chicken, it has absolutely no memory of where it was before the current minute. Living in a very rural area, the probability of being run over by a car or meeting some other mishap is negligible.

1. What is the probability that the chicken is on the north side of the road after one minute? After two minutes?
2. Calculate the probability of being on the north side at each minute up to ten minutes. (The calculation isn't very hard but if you know how you can write a little computer program for it.) Plot the probabilities against time. Repeat, starting the chicken on the south side of the road, and plot on the same graph.

Alternatively, you might be able to derive a formula for the probability of being on the north side after  $t$  minutes (in terms of probabilities of crossing,  $p_S$  and  $p_N$ ).

3. Examining either your formula or your graph, does the probability of being on the north side seem to be approaching a limiting value? What do you think that value is?
4. Now take it as a given that the probability of being on the north side approaches a fixed value (the "equilibrium value") over time. That is, if the probability of being on the north side equals the equilibrium value  $p_E$  at minute  $t$ , it is the same at minute  $t + 1$ . Derive a formula for  $p_E$  in terms of  $p_S$  and  $p_N$ , the probabilities of crossing over from the south or north sides respectively. (Hint: one way of solving this uses the fact that for the probability of being on the north to stay the same, the probability of crossing from north to south must balance the probability of crossing from south to north.)

Sequences of events such as these, in which the probabilities for each event depend on the immediately preceding one but not on earlier events, are called Markov chains. They are widely used in modelling throughout the social and natural sciences. For example, every unemployed person has a certain probability of getting a job, while every employed person has a certain probability of losing their job; somebody who is currently registered as a Democrat has a certain probability of switching to Republican, etc.