Stat 104 - Fall 2004

Assignment 6 **Solution keys**

Written Assignment (40pts total)

5.2 (3pts)

(a) No, the total number of trials is not fixed. **(1pt)**

(b) Yes, the total number of trials is fixed as 100. **(1pt)**

(c) Yes, we have the fixed total number of trials because a year has 52 weeks. **(1pt)**

5.6 (2pts)

(a) From Table C, the answer is 0.0074 because the probability that 11 graduate is the same as the probability that 9 fail to graduate. **(1pt)**

(b) Having 11 or fewer graduate is the same as 9 or more failing to graduate, so the answer is 0.0074+0.0020+0.0005+0.0001=0.01. **(1pt)**

5.12 (4pts)

(a) X follows a binomial distribution because of fixed trial, same probability of success, and independent trials. **(1pt)**

(b) A mean is 200*0.4=80. Using normal approximation, P(75=X=85) = P(74.5<X<85.5) = 0.5727 based on the standard deviation of 6.9282. **(2pts)**

(c) P(X=100) = P(X>99.5) = about 0.002, so we would be suspicious of the 40% figure. **(1pt)**

5.32 (5pts)

(a) P(score=23) = P(Z=0.4255) = 0.3352. **(1pt)**

(b) A mean is 21 and a standard deviation is 4.7/v50 = 0.6647. **(2pts; 1pt each)**

(c) P(mean score=23) = P(Z=3.009) = 0.00131. **(1pt)**

(d) Because the variability of the average is less for larger sample sizes, the normal probability calculation in (c), which has 50 observations, is more accurate. **(1pt)**

5.40 (3pts)

(a) The distribution is approximately normal with a mean of 2.2 and a standard deviation of 0.1941. **(1pt)**

(b) P(average<2) = P(Z<-1.03) = 0.1515. **(1pt)**

(c) P(accidents<100) = P(average<100/52=1.923) = P(Z<-1.427) = 0.0768. **(1pt)**

5.46 (5pts)

(a) The distribution is approximately normal with a mean of 34 and a standard deviation of 2.3534. **(1pt)**

(b) The distribution is approximately normal with a mean of 37 and a standard deviation of 2.2454. **(1pt)**

(c) The distribution of the difference is approximately normal with a mean of 37-34=3 and a standard deviation of v(12^2/26 + 11^2/24) = 3.2527. **(2pts; 1pt each for the mean and sd)**

(d) P(difference>4) = P(Z>0.3074) = 0.3793. **(1pt)**

6.6 (3pts)

(a) 4.5/v24 = 0.9186. **(1pt)**

(b) 95% CI $=(\bar{x}-1.96\sigma_{\bar{x}}, \bar{x}+1.96\sigma_{\bar{x}})=$ (59.99, 63.59kg), so the confidence interval does not contain 65kg, which gives us the evidence against 65kg. **(2pts; CI and interpretation)**

6.22 (2pts)

(a) 98% CI uses the z-value of approximately 2.33. (10.0023-2.33*(0.0002/v5), 10.0023+2.33*(0.0002/v5)) = (10.0021, 10.0025g). **(1pt)**

(b) 2.33*(0.0002/vN) = 0.0001, so N = 21.7. Because N is an integer, N should be 22. **(1pt)**

6.26 (2pts)

(a) $(0.95)^3 = 0.8574$, so 85.74% **(1pt)**

(b) P(covers at least two out of three) = P(fails to cover at most one out of three). Look up n=3, k=0 or 1, and p=0.05 from Table C. Then the probability is 0.8574+0.1354 = 0.9928. **(1pt)**

6.34 (2pts)

(a) $H_0: \mu = \$62,500 \quad vs. \quad H_a: \mu > \$62,500$ **(1pt)**

(b) $H_0: \mu = 2.6 \quad vs. \quad H_a: \mu \neq 2.6$ **(1pt)**

6.48 (3pts)

(a) $H_0: \mu = 9.5 \quad vs. \quad H_a: \mu \neq 9.5$ **(1pt)**

(b) Z = (9.58-9.5)/(0.4/v180) = 2.68. p-value = 2*P(Z>2.68) = 0.0074, which gives the strong evidence for the mean greater than 9.5. **(1pt)**

(c) (9.58-1.96*(0.4/v180), 9.58+1.96*(0.4/v180)) = (9.52, 9.64) **(1pt)**

6.64 (3pts)

(a) (61.79-1.96*(4.5/√24), 61.79+1.96*(4.5/√24)) = (59.99, 63.59) **(1pt)**

(b) Because the 95% CI contains 61.3, we don't reject the null hypothesis. **(1pt)**

(c) Because the 95% CI again contains 63, we don't reject the null hypothesis either. **(1pt)**

6.66 (3pts)

• Because this is a one-side test, p-value is P(z>1.13) = 0.1292. **(1pt)**

• Although there is some difference between two groups, it is small enough to have arisen by chance. Thus, this is not statistically significant. **(1pt)**

• Even when there is truly no difference between two groups, we would still observe the difference of two points in 13% of samples. Thus the author concludes that patents and trade secrets don't help. **(1pt)**

Bonus Problem (10pts total)

1. (8pts)

(a) P(Yes) = P(Yes|H)P(H) + P(Yes|T)P(T) = 1*(1/2) + p*(1/2) = (1+p)/2. **(1pt)**

(b) B(n, (1+p)/2), so the mean, E(X), is n*(1+p)/2. **(1pt)**

(c) X=n*(1+p)/2 gives $\tilde{p}$ =2X/n - 1. **(1pt)**

(d) E($\tilde{p}$)=2E(X)/n - 1 = p, so it is unbiased. **(1pt)**

(e) Var($\tilde{p}$)=$(2/n)^2$ * Var(X) = $(2/n)^2$ * n * (1+p)/2 * (1- (1+p)/2) = $(1-p^2)$/n. **(1pt)**

(f) E($\hat{p}$)=E(Y)/m = p, so it is unbiased. Var($\hat{p}$)=Var(Y)/m^2 = mp(1-p)/m^2 = p(1-p)/m. **(2pts)**

(g) Var($\tilde{p}$)=Var($\hat{p}$), i.e., $(1-p^2)$/n = p(1-p)/m, which gives us

$$n = m\frac{1-p^2}{p(1-p)} = m\frac{1+p}{p} > m$$

Note that this depends on *p*. The smaller *p* is, the bigger the factor that *m* gets multiplied by. In the best case (*p* = 1), *n* = 2*m*. When *p* = 0.5, *n* = 3*m*.

(1pt)

2. (2pts)

(a) X~ B(n, (1+p)/3), so E($\bar{p}$) = 3*E(X)/n-1 = p, so it is unbiased. **(1pt)**

(b) Var($\bar{p}$) = $(3/n)^2$ * Var(X) = $(3/n)^2$ * n * (1+p)/3 * (1-(1+p)/3) = (1+p)*(2-p)/n. **(1pt)**