

1. (10 points) The Governor of a certain state has decided to come out strongly for prison reform and is preparing a new early release program. Its guidelines are fairly simple: if a prisoner is related to a member of the Governor’s staff, he has a 90% chance of being pardoned; if he is not a relative, his chances for release are 0.01. Suppose that 40% of all inmates are related to someone on the Governor’s staff.
 - (a) (4 points) What is the probability that a random selected prisoner will be released?
 - (b) (4 points) Suppose that we know a prisoner is released. What is the probability that this prisoner is related to somebody on the Governor’s staff
 - (c) (2 points) Are whether a prisoner is released and whether a prisoner is a relative of a member of the Governor’s staff independent? Briefly justify.

2. (10 points) A local tavern has 6 bar stools. The bartender predicts that if 2 strangers come into the bar, they will sit in such a way as to leave at least 2 stools between them.
 - (a) (5 points) If 2 strangers do come in, but choose their seats at random, what is the probability that the bartender’s prediction becomes true?
 - (b) (5 points) What is the expected number of seats between the two strangers, assuming that they choose their seats at random?

3. (20 points) For persons in a certain state convicted of grand theft auto and given three-years sentences, the length of time, X , they actually serve is described by the pdf

$$f_X(x) = \frac{1}{9}x^2; \quad 0 \leq x \leq 3$$

- (a) (5 points) What is the probability that a randomly selected prisoner will serve at least 2 years of their sentence.
- (b) (5 points) What is the first quartile (25th percentile) of the time actually served.
- (c) (5 points) Consider a sequence of prisoners who have been released. What is the probability that the third prisoner in the sequence is the first prisoner to have served a term of less than 2 years?
- (d) (5 points) Considering this sequence of prisoners again, what is the expected number of prisoners considered until the first prisoner serving a term of less than 2 years is observed?