Statistics 110 – Practice Quiz 1 Solutions — Summer, 2006

1. (10 points) The Governor of a certain state has decided to come out strongly for prison reform and is preparing a new early release program. Its guidelines are fairly simple: if a prisoner is related to a member of the Governor's staff, he has a 90% chance of being pardoned; if he is not a relative, his chances for release are 0.01. Suppose that 40% of all inmates are related to someone on the Governor's staff.

   (a) (4 points) What is the probability that a random selected prisoner will be released?

$$\begin{aligned} P[\text{Released}] &= P[\text{Released}|\text{Related}]P[\text{Related}] + P[\text{Released}|\text{Not Related}]P[\text{Not Related}] \\ &= 0.9 \times 0.4 + 0.01 \times 0.6 \\ &= 0.366 \end{aligned}$$

   (b) (4 points) Suppose that we know a prisoner is released. What is the probability that this prisoner is related to somebody on the Governor's staff

$$\begin{aligned} P[\text{Related}|\text{Released}] &= \frac{P[\text{Related} \cap \text{Released}]}{P[\text{Released}]} \\ &= \frac{P[\text{Released}|\text{Related}]P[\text{Related}]}{P[\text{Released}]} \\ &= \frac{0.9 \times 0.4}{0.366} \\ &= 0.9836 \end{aligned}$$

   (c) (2 points) Are whether a prisoner is released and whether a prisoner is a relative of a member of the Governor's staff independent? Briefly justify.

   No.

$$P[\text{Related}|\text{Released}] \neq P[\text{Related}]$$

   or

$$P[\text{Released}|\text{Related}] \neq P[\text{Released}|\text{Not Related}]$$

   are two of many ways to justify this.

2. (10 points) A local tavern has 6 bar stools. The bartender predicts that if 2 strangers come into the bar, they will sit in such a way as to leave at least 2 stools between them.

   (a) (5 points) If 2 strangers do come in, but choose their seats at random, what is the probability that the bartenders prediction becomes true?

   There are $\binom{5}{2} = 15$ different ways to allocate the 2 strangers to chairs. The breakdown on the number of seats between them is

Chairs between	0	1	2	3	4
# allocations	5	4	3	2	1

   Therefore

$$P[\geq 2 \text{ seats between}] = \frac{3+2+1}{15} = \frac{2}{5}$$

   (b) (5 points) What is the expected number of seats between the two strangers, assuming that they choose their seats at random?

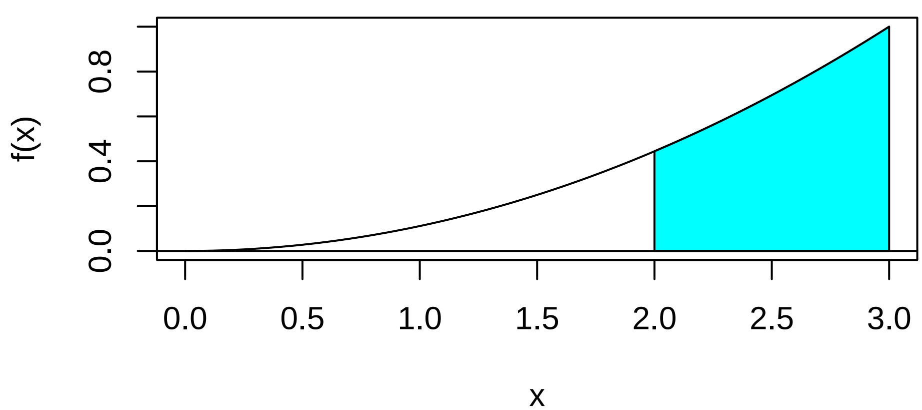
$$E[\text{seats between}] = 0 \times \frac{5}{15} + 1 \times \frac{4}{15} + 2 \times \frac{3}{15} + 3 \times \frac{2}{15} + 4 \times \frac{1}{15} = \frac{20}{15} = 1.33$$

3. (20 points) For persons is a certain state convicted of grand theft auto and given three-years sentences, the length of time, X, they actually serve is described by the pdf

$$f_X(x) = \frac{1}{9}x^2; \qquad 0 \leq x \leq 3$$

   (a) (5 points) What is the probability that a randomly selected prisoner will serve at least 2 years of their sentence.

$$P[X \geq 2] = \int_2^3 \frac{1}{9}x^2 dx = \left.\frac{x^3}{27}\right|_2^3 = 1 - \frac{8}{27} = \frac{19}{27} = 0.7037$$



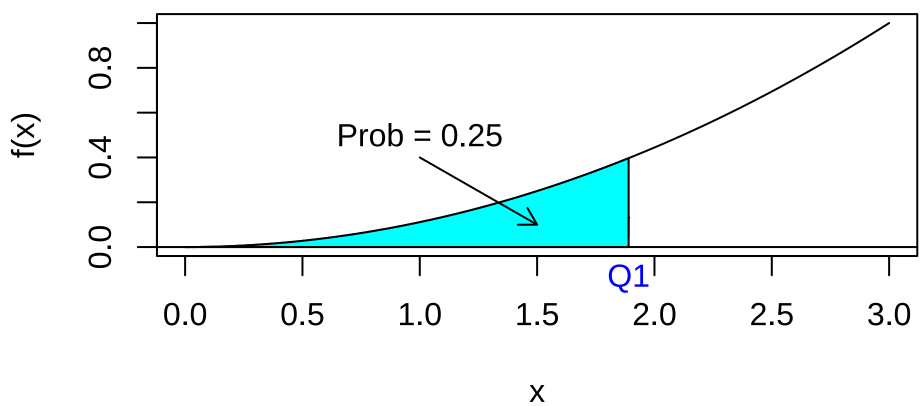
(b) (5 points) What is the first quartile (25th percentile) of the time actually served.

The CDF is

$$F_X(x) = \int_0^x \frac{1}{9}t^2 dt = \left.\frac{t^3}{27}\right|_0^x = \frac{x^3}{27}$$

The first quartile ($Q1$) satisfies

$$F_X(Q1) = \frac{Q1^3}{27} = 0.25$$



Solving for $Q1$ gives

$$Q1 = \sqrt[3]{27 \times 0.25} = \sqrt[3]{6.75} = 1.8899$$

(c) (5 points) Consider a sequence of prisoners who have been released. What is the probability that the third prisoner in the sequence is the first prisoner to have served a term of less than 2 years?

For this to occur with the third prisoner, the first two must have served for more than 2 years (with prob 0.7037 each) and the third one must have served less than 2 years (with prob 1 - 0.7037 = 0.2963), giving a probability of

$$P[Y = 3] = 0.7037^2(1 - 0.7037) = 0.1467$$

(d) (5 points) Considering this sequence of prisoners again, what is the expected number of prisoners considered until the first prisoner serving a term of less than 2 years is observed?

This situation is described by a geometric distribution with $p = 0.2963$, whose mean is $\frac{1}{p} = \frac{1}{\frac{8}{27}} = \frac{27}{8} = \frac{1}{0.2963} = 3.375$