Statistics 110 – Practice Quiz 2 **Summer, 2006**

1. Let $X \sim N(0, 4)$ and $Y|X = x \sim N(x^2, 1)$

   (a) If $x = 4$, what should you predict for Y?

   (b) If you have no information on the actual value of x (i.e. only know $X \sim N(0, 4)$), what should you predict for Y?

   (c) What is $\text{Var}(Y)$? (Note that $\text{Var}(X^2) = 32$)

   (d) Show that $\text{Corr}(X, Y) = 0$.

2. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with distribution function

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

   and let $V_n = \max(X_1, X_2, \ldots, X_n)$

   (a) Find $P[V_n < c]$ for $0 < c < 1$.

   (b) Find $P[V_n > d]$ for $d > 1$.

   (c) Suppose that you are looking for the maximum to be greater than 0.95. How many observations are need so that the probability that this occurs is at least 0.9?

3. Let $Y_1, Y_2, \ldots, Y_n \overset{iid}{\sim} N(2, 4)$.

   (a) How large must n be in order that

$$P[1.9 \leq \bar{Y} \leq 2.1] = 0.99$$

   (b) Assume that we are only willing to assume the $E[Y_i] = 2$ and $\text{Var}(Y_i) = 4$ (The observations may not be normally distributed). How large must n be in this case to satisfy $P[1.9 \leq \bar{Y} \leq 2.1] = 0.99$?