Statistics 110 – Quiz 1 Solutions **Tuesday, July 11, 2006**

1. (5 points) A small town has 80 registered voters, 45 of whom favour a school bond issue. The town treasurer is going to take a poll of 10 registered voters.

   (a) (2 points) How many different selections of 10 people can be made from the 80 registered voters? (Note: a formula/equation describing the number of selections is a satisfactory answer.)

   $$\binom{80}{10} = 1.646 \times 10^{12}$$

   (b) (3 points) What is the probability that the sample selected will include 9 or more favouring the bond issue? (Note: again a formula/equation describing the number of selections is a satisfactory answer.)

   $$\begin{aligned} P[X \geq 9] &= P[X = 9] + P[X = 10] \\ &= \frac{\binom{45}{9}\binom{35}{1} + \binom{45}{10}\binom{35}{0}}{\binom{80}{10}} = 0.02077501 \end{aligned}$$

2. (10 points) In a T maze, a laboratory animal is given a choice of going to the left and getting food or going to the right and receiving a mild electrical shock. Assume that before any conditioning (in trial 1) animals are equally likely to go to the left or to the right. After having received food on a particular trial, the probability of going to the left and right become 0.8 and 0.2, respectively, on the following trial. However, after receiving a shock on a particular trial, the probabilities of going to the left and right are 0.6 and 0.4 respectively. Assume that animals only remember the results of the immediately preceding trial and none of the other trials (e.g. for trial 4, they only remember what happens on trial 3, but not trials 1 and 2).

   (a) (5 points) What is the probability that the animal will turn left on trial number 2?

   Let L_i be the event that the animal turns left of trial i and R_i be the event that the animal turns right on trial i.

   $$P[L_2] = P[L_2|L_1]P[L_1] + P[L_2|R_1]P[R_1] = 0.8 \times 0.5 + 0.6 \times 0.5 = 0.7$$

   (b) (5 points) What is the conditional probability that the animal turned left on trial number 1 given that the animal turned left on trial number 2?

   $$P[L_1|L_2] = \frac{P[L_1 \cap L_2]}{P[L_2]} = \frac{0.8 \times 0.5}{0.7} = \frac{4}{7}$$

3. (25 points) A bombing plane flies directly above a railroad track. Assume that if a large bomb falls within 25 feet of the track, the track will be sufficiently damaged so that traffic will be disrupted. Let X denote the perpendicular distance (in feet) from the track that a bomb falls and assume its density function is

$$f_X(x) = \frac{100 - x}{5000}; \qquad 0 \leq x \leq 100$$

(a) (5 points) What is the probability that a large bomb will disrupt traffic? (Hint: sketch the density and the regions corresponding to disrupted / not disrupted traffic.)

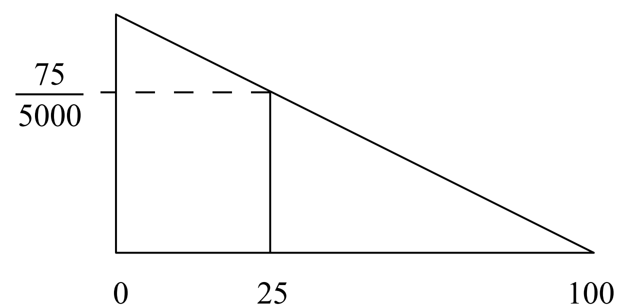
$$F_X(x) = \int_0^x \frac{100 - u}{5000} du = \frac{100x - 0.5x^2}{5000}$$

$$P[X \leq 25] = F_X(25) = 0.4375$$

or

$$P[X \leq 25] = P[X > 25] = 1 - 0.5625 = 0.4375$$

where $P[X > 25]$ is equal to the area of the triangle to the right of 25.

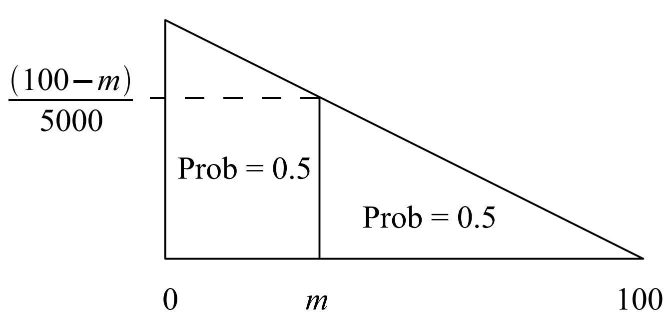


Area $= \frac{75}{5000} \times 75 \times \frac{1}{2} = 0.5625$

(b) (5 points) What is $E[X]$, the expected distance that the bomb falls from the track?

$$E[X] = \int_0^{100} x \frac{100 - x}{5000} dx = \left. \frac{50x^2 - \frac{1}{3}x^3}{5000} \right|_0^{100} = 33.33$$

(c) (5 points) What is the median distance that the bomb falls from the track?
Want m such that $P[X \leq m] = P[X \geq m] = 0.5$. Can either solve $F_X(m) = 0.5$ or find value m such that the area of the triangle to the right of m is 0.5



Area $= \frac{1}{2} \frac{100-m}{5000}(100 - m) = 0.5$

Both cases are equivalent to solving

$$\frac{(100-m)^2}{10000} = \frac{1}{2}$$

which gives $100 - m = \sqrt{5000}$ or $m = 29.29$.

(d) (5 points) If the plane can carry three large bombs and drops all three, what is the probability that traffic will be disrupted? Assume that the impact location of each bomb is independent of the others.

Let X_i be the distance that bomb i falls from the tracks and N be the number of bombs falling within 25 feet of the tracks.

$$\begin{aligned} P[\text{Disrupted}] &= 1 - P[\text{Not Disrupted}] \\ &= 1 - P[N=0] = 1 - (P[X_1 > 25])^3 = 1 - 0.5625^3 = 0.822 \end{aligned}$$

(e) (5 points) Based on the situation of part (d), what is the expected number of bombs that land close enough to the tracks to disrupt traffic?

$N \sim Bin(3, 0.4375)$ so $E[N] = np = 3 \times 0.4375 = 1.3125$, or equivalently

$$E[X] = \sum_{i=0}^{3} i \binom{3}{i} 0.4375^i 0.5625^{3-i} = 1.3125$$