citement and tension that the offer to switch is supposed to create) requires *not* adopting the vos Savant scenario.

Those who are interested may wish to generalize these results by allowing the host the option of immediately opening the player's chosen door. The considerations for the conditional game do not change, but there are other questions to ask in the unconditional game if the outcome of winning immediately is to be taken into account. One could also consider n doors as does vos Savant in the September article, but we do not see, as she seems to, that this offers any particular insight for the case $n = 3$. Indeed, we found that this approach shows mainly that $n = 3$ is the most interesting case. Other generalizations appear to be of less interest. One possibility is to incorporate prior information on the part of the player as to the location of the car, or, related to this, to allow nonuniform probabilities of assignment of the car to the three doors, but these are unlikely to correspond to a real playing of this particular game show situation.

The intricacies of this simple problem make it an excellent teaching tool, as can be seen from the insights offered by the false solutions F1–F6 and the correct resolution. But be forewarned, should your students know the history of this problem, one will invariably complain, "How do you expect me to solve a problem that stumped scores of Ph.D.'s and confused the world's most intelligent person?"!

[*Received January 1991. Revised April 1991.*]

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Comment

Richard G. Seymann*

Morgan, Chaganty, Dahiya, and Doviak, in their solution to the three door game show problem, conclude with the amusing yet valid question of how to respond to the student who, having been assigned the problem, complains, "How do you expect me to solve a problem that stumped scores of Ph.D.'s and confused the world's most intelligent person?" This question deserves a well considered response and is best answered by separating it into two distinct issues. The first is concerned with clarity of problem definition, and the second is concerned with why sensible and mathematically well-trained people, given that they agree on what the problem is, still get the wrong solution. References to both may be found in historical as well as current literature.

The question of problem definition, undoubtedly a complication for those attempting to solve the three door game show problem, is not without its precedents. For example, in 1899 Joseph Bertrand (Weatherford 1982, p. 56) posed a problem that has come to be known as Bertrand's Paradox, and which may be stated as follows: "for a given circle, what is the probability that a random chord is longer than the side of an inscribed equilateral triangle?" Weatherford noted that "At least three different solutions are possible," and continued:

> 1 If one attends to the end-points of the random chord and computes their possible location, the resulting probability is 1/3.
>
> 2 If one attends to the location of the chord's mid-point along the length of the diameter which bisects it, the probability is 1/2.
>
> 3 Finally, if one asks whether the mid-point of the chord does or does not fall within a concentric circle of appropriate diameter, the probability seems to be 1/4.

Without a clear understanding of the precise intent of the questioner, there can be no single correct solution to any problem. Thus, with respect to the three door problem, the answer is dependent on the assumptions one makes about the intent of the one who initially posed the question. Marilyn presented what Morgan et al. call the "vos Savant scenario" and proffered the correct answer. Simply put, and quite clear considering her suggestions for simulation procedures in her two later columns, the host is to be viewed as nothing more than an agent of chance who *always* opens a losing door, reveals a goat, and offers the contestant the opportunity to switch to the remaining, unselected door. "Anything else is a different question," she says. That she didn't offer a rigorous, mathematical proof in a popular Sunday supplement does her no discredit. Perhaps some of Marilyn's readers did suspect an alternate game in which the host has an ulterior motive, but none of the published letters even hinted at this possibility. Unfortunately, not being clairvoyant, it is impossible to know what assumptions they may have been operating under. It is reasonable to infer, however, from the bold certainty of their statements, that either they understood precisely the intent of the question and simply got it wrong or that they recognized the further complexities and still got it wrong.

*Richard G. Seymann is Professor of Statistics and Business Administration, School of Business, Lynchburg College, Lynchburg, VA 24501.

Morgan and his colleagues consider both alternatives which are, to use game theory terminology, whether the contestant is involved in a one-player game or a two-player game. Marilyn presents it as the former, while Morgan et al., though apparently recognizing the validity of Marilyn's view, emphasize the latter, comparing the game show problem with the prisoner's dilemma and assuming that both the host and the contestant have something to gain. From the point of view of the professional probabilist, this is certainly the more mathematically interesting of the two alternatives and results in a solution that ultimately satisfies both scenarios.

This leads us to the second part of the student's question. If all, or even most, of those Ph.D.'s thought the question was concerned with the simpler problem, and if they are, as certainly seems reasonable, sensible and mathematically well-trained people, why did they get the solution wrong? The most obvious answer is that the solution, even in the one-player game scenario, is counter-intuitive. History is replete with similar instances. One need only recall Bertrand's Box Problem (not to be confused with the previously cited Bertrand's Paradox), wherein a box has three drawers, one containing two gold coins, one containing two silver coins, and one containing one gold and one silver coin. If one drawer is "randomly" chosen, and if a coin that is "randomly" selected from that drawer turns out to be gold, what is the probability that the chosen drawer is the one containing two gold coins? The answer, as in the three door problem, is not 1/2, which immediately appeals to intuition, but 2/3. Consider also D'Alembert, a distinguished mathematician of the eighteenth century, who believed the probability of obtaining a head in two tosses of a fair coin to be 2/3 by reasoning that the sample space contained only the three outcomes, HH, TH, and TT.

Most of us are familiar with "The Birthday Problem" (Feller 1957, p. 33) that leads to the "astounding" and counter-intuitive result that, for as small an aggregate as 23 people, the probability that at least two share a common birthday is greater than one-half. There is also the corollary birthday problem that requires an aggregate of at least 253 (not 183) people to be more than 50% certain that at least one of them shares my birthday (January 6).

More recently there have been numerous articles (compiled by Kahneman, Slovik, and Tversky 1982) indicating that, when making judgments regarding the likelihood of uncertain events, even mathematically sophisticated people do not follow the principles of probability theory. "This conclusion is hardly surprising because many of the laws of chance are neither intuitively apparent, nor easy to apply" (p. 32). Included are many examples that, if carefully analyzed, are straightforward enough, yet have results that do not conform to one's initial, intuitive estimate. Among those examples is the following question presented to 60 students and staff at the Harvard Medical School.

> If a test to detect a disease whose prevalence is 1/1000 has a false positive rate of 5%, what is the chance that a person found to have a positive result actually has the disease, assuming you know nothing about the person's symptoms or signs?

Over half the participants responded 95%, the "average" response was 56%, and only 11 gave the correct response, 2% (p. 154).

We all have our own favorite anecdotes of the difficulties that ensue when questions are not clearly posed and of those probability problems that yield counter-intuitive results. These anecdotes are what can best provide a foundation for our response to the student's complaint.

One more thing. A surprisingly large number of undergraduate students, when faced with the inescapable conclusion that switching will double their chance of winning, still maintain they wouldn't switch. Their reason for this unexpected decision may be summarized by the statement, "If I switched and lost, I'd kick myself for having switched." As the old professor in *The Chronicles of Narnia* was wont to say of the Pevensie children, "I wonder what they *do* teach them at these schools" (Lewis 1950).

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